Online Advertising

Pricing Models

- CPM (Cost per thousand impressions)
- CPC (Cost per click)
- CPA (Cost per acquisition)
- Conversion rates:
  - Click-through-rate (CTR), conversion from clicks to acquisitions, ...

Differences between these pricing models:

- Uncertainty in conversion rates:
  - Sparse data, changing rates, ...
- Stochastic fluctuations:
  - Even if the conversion rates were known exactly, the number of clicks/conversions would still vary, especially for small samples
Sponsored Search Auction

Advertiser \( \text{Bid} = \text{Cost per Click} \ C \) Auctioneer (Search Engine) CTR estimate \( Q \)

- Value/impression ordering: \( C_1 Q_1 > C_2 Q_2 > \ldots \)
- Give impression to bidder 1 at CPC = \( C_2 Q_2 / Q_1 \)

VCG Mechanism: Truthful for a single slot, assuming static CTR estimates Can be made truthful for multiple slots [Vickrey-Clark-Groves, Myerson81, AGM06]

This talk will focus on single slot for proofs/examples
When Does this Work Well?

- High volume targets (keywords)
  - Good estimates of CTR

- What fraction of targets are high volume?
  - Folklore: a small fraction
  - **Motivating problem:**
    - How to better monetize the low volume keywords?
**Traffic Estimator**

- Average CPC: $0.00 (at a maximum CPC of $0.05)  
  Estimated clicks per day: 0 (at a daily budget of $1.00)

Estimates are based on your bid amount and geographical targeting selections. Because the Traffic Estimator does not consider your daily budget, your ad may receive fewer clicks than estimated.

<table>
<thead>
<tr>
<th>Keywords</th>
<th>Search Volume</th>
<th>Estimated Avg. CPC</th>
<th>Estimated Ad Positions</th>
<th>Estimated Clicks / Day</th>
<th>Estimated Cost / Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>living trust bay area</td>
<td></td>
<td>$0.00</td>
<td>Not enough data to give estimates</td>
<td>0</td>
<td>$0</td>
</tr>
</tbody>
</table>

**Notes about these estimates for your keywords and targeting:**
- Because your campaigns do not yet have a performance history, keyword estimates are based on system-wide performance information.
- We have too little data to estimate traffic for your selections. Try adding keywords or choosing more languages or a larger target area.
Possible Solutions

- Coarse ad groups to predict CTR:
  - Use performance of advertiser on possibly unrelated keywords

- Predictive models
  - Regression analysis(feature extraction)
  - Taxonomies/clustering
  - Collaborative filtering

- **Our approach**: Devise richer pricing models
Hybrid Scheme

Bid$_1$ = Cost per Impression
Bid$_2$ = Cost per Click

<$M, C$>

Advertiser               Auctioneer (Search Engine)
Hybrid Scheme

Bid_1 = Cost per Impression
Bid_2 = Cost per Click

< M, C >

CTR estimate

Q

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Hybrid Scheme

Advertiser

\[ \text{Bid}_1 = \text{Cost per Impression} \]
\[ \text{Bid}_2 = \text{Cost per Click} \]

\( <M, C> \)

Auctioneer (Search Engine)

\( \text{CTR estimate} \)

\( Q \)

- Advertiser’s score \( R_i = \max \{ M_i, C_i Q_i \} \)
**Hybrid Scheme**

- **Bid** 1 = Cost per Impression
- **Bid** 2 = Cost per Click

- **Advertiser**

  - **<M, C>**

- **Auctioneer** (Search Engine)

  - **CTR estimate**
    - $Q$

- **Advertiser’s score** $R_i = \max \{ M_i, C_i Q_i \}$

- **Order by score:** $R_1 > R_2 > ...$
**Hybrid Scheme**

Bid$_1$ = Cost per Impression  
Bid$_2$ = Cost per Click  

Advertiser  

Auctioneer (Search Engine)  

CTR estimate  

$Q$

- Advertisement’s score $R_i = \max \{ M_i, C_i Q_i \}$
- Order by score: $R_1 > R_2 > ...$
- Give impression to bidder 1:
  - If $M_1 > C_i Q_i$ then charge $R_2$ per impression
  - If $M_1 < C_i Q_i$ then charge $R_2 / Q_i$ per click
Why Such a Model?

- Per-impression bid:
  - Advertiser’s estimate or “belief” of CTR
  - May or may not be an accurate reflection of the truth
  - Backward compatible with cost-per-click (CPC) bidding
**Why Such a Model?**

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- **Why would the advertiser know any better?**
  - Advertiser aggregates data from various publishers
  - Has domain specific models not available to auctioneer
  - Is willing to pay a premium for internal experiments
Benefits

1. **Search engine:**
   - Better monetization of low volume keywords

2. **Advertiser:**
   - Opportunity to make the search engine converge to the correct CTR estimate *without paying a premium*

3. **Technical:**
   a) Truthful
   b) Accounts for risk characteristics of the advertiser
   c) Allows users to implement complex strategies
Multiple Slots

- Show the top $K$ scoring advertisers
  - Assume $R_1 > R_2 > ... > R_K > R_{K+1} ...$

- Generalized Second Price (GSP) mechanism:
  - For the $i^{th}$ advertiser, if:
    - If $M_i > Q_i C_i$ then charge $R_{i+1}$ per impression
    - If $M_i < Q_i C_i$ then charge $R_{i+1} / Q_i$ per click
**Multiple Slots**

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- Can also implement VCG
  - Need separable CTR assumption
  - Details in the paper

[Vickrey-Clark-Groves, Myerson81, AGM06]
Bayesian Model for CTR

True underlying CTR = p

Advertiser

Auctioneer (Search Engine)
**Bayesian Model for CTR**

True underlying CTR = \( p \)

- **Advertiser**
  - Prior distribution \( P_{adv} \) (Private)

- **Auctioneer (Search Engine)**
  - Prior distribution \( P_{auc} \) (Public)
Bayesian Model for CTR

True underlying CTR = p

Per-impression bid $M$

Prior distribution $P_{\text{adv}}$
(Private)

Prior distribution $P_{\text{auc}}$
(Public)

CTR estimate $Q$

Advertiser

Auctioneer (Search Engine)
Bayesian Model for CTR

True underlying CTR = p

Per-impression bid $M$

Prior distribution $P_{adv}$ (Private)

Prior distribution $P_{auc}$ (Public)

CTR estimate $Q$

Advertiser

Auctioneer (Search Engine)

Each agent optimizes based on its current “belief” or prior:
Beliefs updated with every impression
Over time, become sharply concentrated around true CTR
What is a Prior?

- Simply models asymmetric information
  - Sharper prior $\Rightarrow$ More certain about true CTR $p$
  - $E[\text{Prior}]$ need not be equal to $p$

- Main advantage of per-impression bids is when:
  - Advertiser’s prior is sharper than auctioneer’s
  - Limiting case: Advertiser certain about CTR $p$

- Priors are only for purpose of analysis
  - Mechanism is well-defined regardless of modeling assumptions
Truthfulness

- Advertiser assumes CTR follows distribution $P_{adv}$
- Wishes to maximize expected profit at current step
  - $E[P_{adv}] = x = \text{Expected belief about CTR}$
  - Utility from click = $C$
  - Expected profit = $C \times x - \text{Expected price}$
Truthfulness

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Let $C_y$ = Per impression bid
$R_2$ = Highest other score
If $\max(C_y, C \times Q) < R_2$ then Price = 0
Else:
  - If $y < Q$ then: Price = $x \times R_2 / Q$
  - If $y > Q$ then: Price = $R_2$
Truthfulness

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- Wishes to maximize expected profit at current step
  - $E[P_{adv}] = x =$ Expected belief about CTR
  - Utility from click = $C$
  - Expected profit = $C \cdot x$ - Expected price

Bidding $(Cx, C)$ is the dominant strategy

Regardless of $Q$ used by auctioneer
Regardless of $P_{adv}$ and true CTR $p$

Elicits advertiser’s “expected belief” about the CTR!
Holds in many other settings (more later)
Conjugate Beta Priors

- \( P_{auc} \) for advertiser \( i = Beta(\alpha, \beta) \)
  - \( \alpha, \beta \) are positive integers
  - Conjugate of Bernoulli distribution (CTR)
  - Expected value = \( \frac{\alpha}{\alpha + \beta} \)

- Bayesian prior update:
  - Probability of a click at the next step is: \( \frac{\alpha}{\alpha + \beta} \)
  - If click, new \( P_{auc} \) (posterior) = \( Beta(\alpha+1, \beta) \)
  - If no click, new \( P_{auc} \) (posterior) = \( Beta(\alpha, \beta+1) \)
Evolution of Beta Priors

Click

No Click

Denotes Beta(1, 1)
Uniform prior
Uninformative

\[ \text{E}[P_{\text{auc}}] = \frac{1}{4} \]

\[ \text{E}[P_{\text{auc}}] = \frac{2}{5} \]

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Properties

- Larger $\alpha$, $\beta \Rightarrow$ Sharper concentration around $p$
  - Uninformative prior: $Beta(1,1) = Uniform[0,1]$

- $Q = \mathbb{E}[P_{auc}] = \frac{\alpha}{(\alpha + \beta)}$
  - Encodes auctioneer’s “belief”
  - Could be different from true CTR $p$
Certain Advertiser

- Knows true CTR $p$ and bids rationally $(M_i = p_i)$
  - $P_{adv} = p_i$ with probability 1
  - $P_{auc} = \text{Beta}(\alpha_i, \beta_i)$ and $Q_i = \mathbb{E}[P_{auc}] = \alpha_i / (\alpha_i + \beta_i)$

- Revenue properties of auctioneer:
  - Worst case: 63% of CPC scheme
  - Canonical case: $\log n$ times better than CPC scheme

- Flexibility for advertiser:
  - Can make $P_{auc}$ converge to $p$ without losing revenue
  - But pays huge premium for achieving this in CPC auction
Better Monetization

- Illustrative Scenario: Low volume keywords
  - $n$ advertisers, all click-utilities $C = 1$
  - All $P_{\text{auc}} = \text{Beta}(1, \log n)$ so that $\mathbb{E}[P_{\text{auc}}] = Q \approx 1 / \log n$
    - High variance prior
    - Some $p_i$ close to 1 with high probability
  - Per-impression bid will elicit this high $p_i$
  - CPC auction allocates slot to a random advertiser

- **Theorem**: Hybrid auction can generate $\log n$ times more revenue for auctioneer than existing CPC auction
Flexibility for Advertisers

- Suppose advertiser certain about CTR = \( p \)
  - Assume \( C = 1 \) and \( Q < p \)
  - Bids truthfully and wins on per impression bids

- Hybrid scheme: Charged at most \( p \) per impression
  - Impressions shown repeatedly
  - Auctioneer’s belief \( P_{auc} \) will converge to have mean \( p \)
  - Now, advertiser switches to CPC bidding

- Assume auctioneer’s prior is \( Beta(\alpha, \beta) \)
  - \( Q = \alpha / (\alpha + \beta) < p \)
Flexibility for Advertisers

- If CTR converges in $T$ impressions resulting in $N$ clicks:
  - $(\alpha + N)/(\alpha + \beta + T) \geq p$
  - Since $Q = \alpha/(\alpha + \beta) < p$, this implies $N \geq Tp$
  - Value gain = $N$; Payment for $T$ impressions at most $T \times p$
  - Hence, no loss in revenue to advertiser!

- In the existing CPC auction:
  - The advertiser would have to pay a huge premium for getting impressions and making the CTR converge
Uncertain Advertisers

- Advertiser should “pay premium” for CTR $p$ resolving to a high value
  - What should her bidding strategy be?
  - Does it lead to a socially optimal mechanism?

- Key contribution:
  - Defining a Bayesian model for repeated auctions
  - Dominant strategy exists!
**Semi-Myopic Advertiser**

- Maximizes discounted utility in contiguous time horizon in which she wins the auction
  - State of other advertisers stays the same during this time
  - Once she stops getting impressions, cannot predict future
    ... since future will depend on private information of other bidders!
  - Circumvents negative results in economics literature

- Private information with advertiser:
  - Discount factor $\gamma$, value $C_i$ and prior $P_{adv}$
  - Discount factor models varying optimization horizons
    - Strategic vs. myopic
Dominant Hybrid Strategy

- Bidder always has a dominant hybrid strategy
  - **Bidding Index**: Computation similar to the Gittins index
  - Bidder can optimize her utility by dynamic programming
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- **Socially optimal** in many reasonable scenarios:
  - **Myopic advertiser**: Has $\gamma_i = 0$; trusts auctioneer’s prior:
    - Pure per-click bidding implements the Gittins index policy
  - If advertiser is *certain* of CTR, and $Q_i$ is an underestimate:
    - Bidding index = Per-impression bidding, which is socially optimal
  - Implementation needs *both* per-impression and per-click bids
Summary

- Allow both per-impression and per-click bids
  - Same ideas work for CPM/CPC + CPA
- Significantly higher revenue for auctioneer
- Easy to implement
  - Hybrid advertisers can co-exist with pure per-click advertisers
  - Easy path to deployment/testing
- Many variants possible with common structure:
  - Optional hybrid bids
  - Use the “max” operator to compute score
Open Questions

- Some issues that may be exacerbated:
  - Whitewashing: Re-entering when CTR is lower than the default
  - Fake Clicks: Bid per impression initially and generate false clicks to drive up CTR estimate $Q$
    - Switch to per click bidding when slot is “locked in” by the high $Q$

- Analysis of semi-myopic model
  - Other applications of separate beliefs?

- Connections of Bayesian mechanisms to:
  - Regret bounds and learning [Nazerzadeh, Saberi, Vohra ‘08]
  - Best-response dynamics [Edelman, Ostrovsky, Schwarz ‘05]